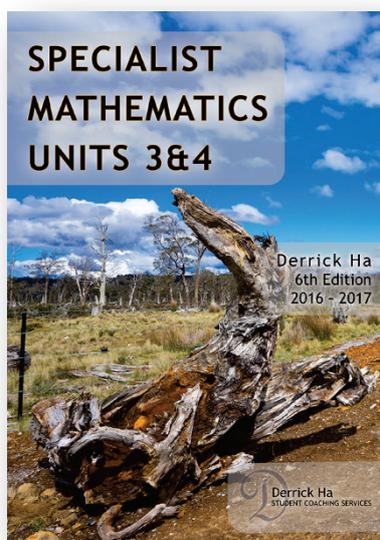


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# Specialist Mathematics Units 3&4

## 6<sup>th</sup> Edition



264 pages  
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- About the Author
- Sample – Theory
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# Preface

In writing this book, it is my hope that you, the student, will discover a new perspective on mathematics. My book aims to complement your teacher's guidance and your regular school textbook by providing a different viewpoint on every topic – the unique viewpoint of a former student who has had success in the VCE and knows the intricacies of what is required. In addition, I hope to share with you the insights I have gained from teaching and lecturing in VCE mathematics over the last decade.

Most study guides will try to provide you with hundreds or thousands of 'exam style questions'. It is easy to be drawn towards such books. However, without a true understanding of the mathematical concepts, such books are rather useless.

I feel that it is more important to teach students why things work, rather than just showing them thousands of examples that they do not properly understand. This idea has formed the basis for my teaching and mathematics books over the last nine years. My aim has always been to explain concepts in a simple and easy-to-understand manner, and to provide students with quality questions rather than a large quantity of questions. It is with this approach that I am proud to bring you this sixth edition of my mathematics guides.

In this book, you will find practical explanations of each concept to address the questions that you are likely to have when learning the material. You will also find advice that is specific to the VCE, and which will assist you in targeting your learning towards SACs and the end-of-year exams. Other parts of this book will improve your understanding by delving into the complexities of each topic so that you can answer the more difficult analysis questions that will inevitably appear in your assessments.

To use this book most effectively, I suggest that you firstly read through it in the same way as you would for a novel – that is, from beginning to end with as few breaks as possible. I have written in a conversational style to create a more interesting read, as well as to provide you with descriptions that are easier to relate to and understand, especially when compared with a regular school text book.

One of my other motivations for writing this book stems from my time as a school student. During my VCE, I found that there were no resources available that showed how to set out a solution for a VCE mathematics exam. The textbooks were not handwritten, and neither were the official VCAA assessment reports. As such, I hope to fill this gap, and have included many hand-written examples to demonstrate the layout of solutions for typical exam questions. These solutions show you the same format that I used during my VCE to much success, and you can use these hand-written answers as a guide for setting out your own working.

The solutions also feature annotations that illustrate the thinking that I use to write an answer. I am sure that you will have encountered many situations where you understand the solution provided, but do not know why or how to come up with such a solution. The aim of my annotations is to tell you the thought process behind a solution and what I am thinking

when I write. This will help to improve your ability to think on your feet and guide your thoughts when tackling more difficult questions.

When you read through this book, you will also encounter many examples. For the lengthier examples, you should attempt the question before looking at the annotated solution. This will allow you to critique your working, which is important because many students unwittingly lose marks and waste time as a result of incorrect or disorganised 'working out'.

As you know, revision throughout the year is very important, and I am sure that many of you will not revise as often as you should. Therefore, I have placed a set of revision questions after every few chapters. These questions are written in exactly the same style as you will encounter in the end-of-year exams, and so will help you to adjust to the unique format of VCE exam questions. They will also assist you with beginning your exam preparation earlier in the year. For these exam questions, I provide an indication of the difficulty level so that you can measure your progress, especially in terms of where you need to be to succeed in your final exams.

I have also included four Trial Exams for later in the year. With the sheer number of exams available to students from both VCAA and other companies, I feel that it would be pointless for me to write another 'standard exam'. There are already many easy and 'standard' exams available, and you should definitely attempt these first when you begin your exam preparation. However, you will eventually reach a stage when you want to extend yourself to prepare for those more difficult VCE exam questions. It is then that you will be ready to tackle the trial exams that I have written.

My trial exams relate directly to the new VCE study design for 2016, but I have deliberately written questions that are extremely difficult in nature. My exams contain all the tricks and traps that I can think of, and are based on my own experiences as a VCE student and a teacher. Don't be disheartened by unsuccessful attempts at my exams – rather, learn from them, so that you gain valuable insight into the potential pitfalls when attempting similar questions in the future.

Finally, at the end of this book, you will find space for your own notes. I have written this book with you, the student, in mind. You will need a bound reference book for your SACs and exams, and it would be a waste of time for you to write your own notes. I want you to use this book as your reference book, but understand that you may also want to add your own notes. This space will allow you to do so.

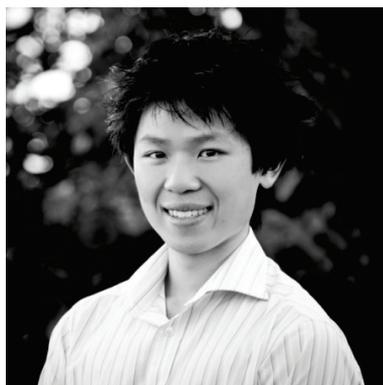
For now, begin reading, writing and working through this student guide. If you can absorb all of the concepts, tips and suggestions, I am sure that you will soon be on a path towards mathematical success.

I wish you all the best for your studies.

Derrick Ha  
September 2016

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# About the Author



Over the past nine years, Derrick has established himself as an author, tutor and lecturer in senior VCE mathematics. Since founding Derrick Ha Student Coaching Services in 2008, Derrick and his team have assisted thousands of students with their VCE and helped them to achieve their goals.

Derrick's unique teaching style is drawn from his personal experiences of the VCE and his active extracurricular involvement. In 2007, Derrick attained the top ENTER (ATAR) score of 99.95, with the following results:

	<b>Subject</b>	<b>Raw Study Score</b>	<b>Year</b>
<b>Top 4 Subjects</b>	English Language	50	2007 (Year 12)
	Specialist Mathematics	50 (scaled to 54.2)	2007
	Accounting	50	2006 (Year 11)
	Mathematical Methods	50	2005 (Year 10)
<b>5th Subject</b>	Chemistry	50 (5.0 increment)	2006
<b>6th Subject</b>	University Mathematics	High Distinction (5.5 increment)	2007
		<b>Aggregate = 214.7</b>	
<b>Extra 7th Subject</b>	English	48	2007

Derrick's achievements also extend outside of the VCE. He had many successes in mathematics competitions and, in 2007, achieved a perfect score in the Australian Mathematics Competition. He was awarded a gold medal and the B H Neumann Certificate for being the only Senior student in Australasia to achieve this perfect score. He was also an invited attendee of four training selection schools for the Australian Mathematical Olympiad Team. In both 2004 and 2005, Derrick was awarded a Diploma by the Russian Academy of Sciences for his accomplishments in the International Tournament of Towns Mathematics Competition.

Derrick also has extensive experience in public speaking and mathematics coaching. He is a motivational speaker for high schools and community organisations, and was a lecturer for state-wide end-of-year revision lectures in both Mathematical Methods CAS and Specialist Mathematics. Derrick also works as a mathematics tutor, and has previously volunteered to teach English and mathematics to Sudanese immigrants.

His accomplishments as an orator include being a speaker in the team that reached the DAV Debating State Finals in four separate years. He also experienced success in mock-law courts, as a speaker in the legal team that won the State Mooting Titles in the 2007 Bond University Mooting Competition.

In his VCE year, Derrick was the School Vice-Captain of Haileybury College and the Firsts Team Badminton Captain. He was a recipient of the VCE Premier's Award Top All-Round Achiever, the Australian Student Prize and the Monash University Prize for Academic Excellence in Year 11.

In 2013, Derrick completed his medical training at the University of Melbourne. He was the recipient of the National Medicine Full Scholarship and won multiple Dean's Honours Awards. Derrick currently works as a full-time medical doctor.

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# Sample – Theory

This is an example of the notes that are provided in the book. These are designed to explain each topic in a practical and easy-to-understand manner. Notes are provided for every topic on the study design.

## Sample Means

The sample mean is a relatively simple concept – it is the mean of the sample. It is important to understand how it works, so we shall consider an example.

**Question:**

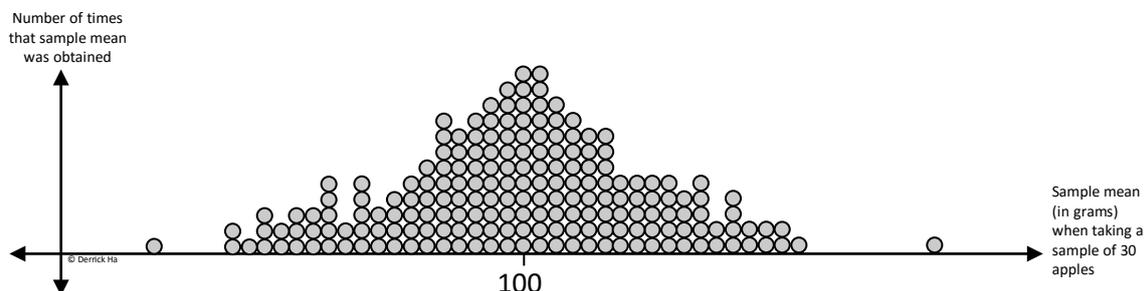
*The mass of the apples from the Carovale Farm is normally distributed with an average mass of 100 grams and a standard deviation of 20 grams. Carolyn goes to the farm and picks a random sample of 30 apples from the trees.*

- a) *Consider the sampling distribution of the sample means with  $n=30$ . State the mean and standard deviation of the sample mean distribution.*
  - b) *Find the probability that the sample has an average mass greater than 100 grams.*
  - c) *Find the probability that the sample has an average mass greater than 110 grams.*
- Give your answer correct to three decimal places.*

Before we delve into the answer, we will first discuss the concepts behind the question. Clearly, when Carolyn takes a random sample of 30 apples, it will depend on how 'lucky' she is. By chance, she might pick a sample of 30 really large apples. She will then have a sample with an average that is much larger than 100 grams. By similar reasoning, she could be 'unlucky' and pick a sample of 30 really small apples. She will then have a sample of 30 apples that have an average that is much less than 100 grams. However, as you can imagine, it would be rather uncommon for either of the above scenarios to happen. It would be much more likely that Carolyn picks a sample of 30 apples that has an average mass that is close to 100g.

Now pretend that one hundred people (or more) came to the farm and randomly took 30 apples each. Clearly some of these people will be lucky and others will be unlucky. However, on average, we would expect that the 'average person' picks a sample of 30 apples that have an average mass of 100g.

Let's now take the 100 sample means for these 100 people. If we plotted these sample means, we could end up with a diagram such as:



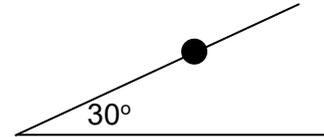
This diagram shows the sampling distribution of the standard mean. In other words, each dot represents the result from one of the people that went to the farm. The dot shows the average mass of the 30 apples for that person. You can see that there are a couple of outliers – one person was really 'lucky' and had a bag of 30 really heavy apples, whilst another had a bag of really light apples. However, most people had 'average bags', in which the mean weight of the 30 apples in the bag was close to 100g... *(Continued in the book)*...

*The remainder of this topic has been omitted*

# Sample – Hand-Written Solution

This is an example of the hand-written solutions provided in the book. These are designed to illustrate Derrick's thought process, as well as showing how you can set out your own working in the VCAA exam.

**Question:** A 3 kg mass is placed on a smooth slope as shown. A pushing force of 5N is also exerted on the particle. This pushing force is parallel to the slope and pushing the particle down the slope. Find the acceleration of the mass (to 2 d.p.), and determine how far it has travelled from rest after 3 seconds.



1

2

4

$$\underline{R} = m\underline{a}$$

// to slope!

$$5 + 3g \sin(30^\circ) = 3a$$

$$\therefore a = \frac{5}{3} + \frac{g}{2}$$

$u = 0, t = 3$

5

$$s = ut + \frac{1}{2}at^2$$

$$\therefore s = 0 + \frac{1}{2}\left(\frac{5}{3} + \frac{g}{2}\right)(9)$$

$$= 29.55$$

$\therefore$  Accel =  $6.57 \text{ ms}^{-2}$  and Distance =  $29.55 \text{ m}$

1 – Again, always begin your working with a force diagram. Show the units. Make sure that you define the direction of acceleration – this is usually done with a double arrow (as shown).

2 – The working which proves that this is  $30^\circ$  is never shown (it is unnecessary). However, you must understand how to work it out. Quite simply, use the properties of a right angled triangle:

Angles in a triangle sum to  $180^\circ$ .

This dotted line is an extension of the normal, and so is perpendicular to the slope.  $60^\circ + 30^\circ = 90^\circ$

3 – Here, I have resolved the weight force into two components – one parallel to the slope and one perpendicular to the slope. We do this so that we can 'isolate' the forces acting parallel to the slope.

4 – I state this equation immediately after my force diagram because it is the basis of the rest of my working.

5 – Refer to the previous section on 'Constant Acceleration' for an explanation of this working.

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# Sample – Exam Question

This is a question from one of the trial exams in the book. Although not shown in this sample, the book also contains hand-written solutions for all of the exams in order to demonstrate how to set out working in a VCE exam.

*(Continued on next page)*

SAMPLE

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*The initial prompt and parts a to d of the following question have been omitted:*

After getting his suitcase, Bryan returns to the ground floor and then to the car park so that he can begin his drive home. He has been working for 30 years and knows that the time that he takes to drive between home and work can be represented by a normally distributed variable with a mean of 25 minutes and a standard deviation of 2 minutes.

- e. What is the probability that he takes longer than 30 minutes to return home from work? Give your answer correct to 3 decimal places.

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1 mark

Bryan likes to plan his car trips so that he can make sure he has enough petrol for the week. One full tank of petrol allows him to drive for a total of 260 minutes. Bryan fills up his petrol tank every Sunday before working for the five days from Monday to Friday. He only uses his car for travelling between work and home.

- f. If Bryan drives to and from work five days a week, then what is the probability that he will be able to last until the end of the week without needing to refill his petrol tank? Give your answer correct to 3 decimal places.

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SAMPLE

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© Derrick Ha

3 marks

One of the fuel injectors in Bryan's car becomes faulty and results in his car consuming more fuel than usual. He now has only a 70% chance of lasting the week without refuelling.

- g. Calculate the value of  $k$ , where  $k$  is the time in minutes that the petrol tank is now able to last. Give your answer correct to the nearest second.

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3 marks