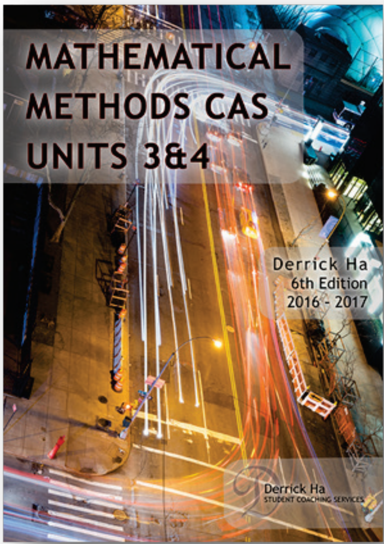
Mathematical Methods CAS Units 3&4

6^{th} Edition



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Preface

In writing this book, it is my hope that you, the student, will discover a new perspective on mathematics. My book aims to complement your teacher's guidance and your regular school textbook by providing a different viewpoint on every topic – the unique viewpoint of a former student who has had success in the VCE and knows the intricacies of what is required. In addition, I hope to share with you the insights I have gained from teaching and lecturing in VCE mathematics over the last decade.

Most study guides will try to provide you with hundreds or thousands of 'exam style questions'. It is easy to be drawn towards such books. However, without a true understanding of the mathematical concepts, such books are rather useless.

I feel that it is more important to teach students why things work, rather than just showing them thousands of examples that they do not properly understand. This idea has formed the basis for my teaching and mathematics books over the last nine years. My aim has always been to explain concepts in a simple and easy-to-understand manner, and to provide students with quality questions rather than a large quantity of questions. It is with this approach that I am proud to bring you this sixth edition of my mathematics guides.

In this book, you will find practical explanations of each concept to address the questions that you are likely to have when learning the material. You will also find advice that is specific to the VCE, and which will assist you in targeting your learning towards SACs and the end-of-year exams. Other parts of this book will improve your understanding by delving into the complexities of each topic so that you can answer the more difficult analysis questions that will inevitably appear in your assessments.

To use this book most effectively, I suggest that you firstly read through it in the same way as you would for a novel – that is, from beginning to end with as few breaks as possible. I have written in a conversational style to create a more interesting read, as well as to provide you with descriptions that are easier to relate to and understand, especially when compared with a regular school text book.

One of my other motivations for writing this book stems from my time as a school student. During my VCE, I found that there were no resources available that showed how to set out a solution for a VCE mathematics exam. The textbooks were not handwritten, and neither were the official VCAA assessment reports. As such, I hope to fill this gap, and have included many hand-written examples to demonstrate the layout of solutions for typical exam questions. These solutions show you the same format that I used during my VCE to much success, and you can use these hand-written answers as a guide for setting out your own working.

The solutions also feature annotations that illustrate the thinking that I use to write an answer. I am sure that you will have encountered many situations where you understand the solution provided, but do not know why or how to come up with such a solution. The aim of my annotations is to tell you the thought process behind a solution and what I am thinking when I write. This will help to improve your ability to think on your feet and guide your thoughts when tackling more difficult questions.

When you read through this book, you will also encounter many examples. For the lengthier examples, you should attempt the question before looking at the annotated solution. This will allow you to critique your working, which is important because many students unwittingly lose marks and waste time as a result of incorrect or disorganised ‘working out’.

As you know, revision throughout the year is very important, and I am sure that many of you will not revise as often as you should. Therefore, I have placed a set of revision questions after every few chapters. These questions are written in exactly the same style as you will encounter in the end-of-year exams, and so will help you to adjust to the unique format of VCE exam questions. They will also assist you with beginning your exam preparation earlier in the year. For these exam questions, I provide an indication of the difficulty level so that you can measure your progress, especially in terms of where you need to be to succeed in your final exams.

I have also included four Trial Exams for later in the year. With the sheer number of exams available to students from both VCAA and other companies, I feel that it would be pointless for me to write another ‘standard exam’. There are already many easy and ‘standard’ exams available, and you should definitely attempt these first when you begin your exam preparation. However, you will eventually reach a stage when you want to extend yourself to prepare for those more difficult VCE exam questions. It is then that you will be ready to tackle the trial exams that I have written.

My trial exams relate directly to the new VCE study design for 2016, but I have deliberately written questions that are extremely difficult in nature. My exams contain all the tricks and traps that I can think of, and are based on my own experiences as a VCE student and a teacher. Don’t be disheartened by unsuccessful attempts at my exams – rather, learn from them, so that you gain valuable insight into the potential pitfalls when attempting similar questions in the future.

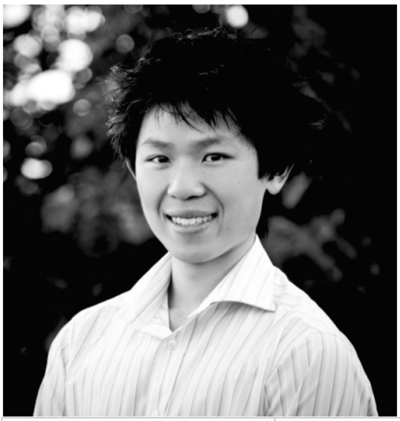
Finally, at the end of this book, you will find space for your own notes. I have written this book with you, the student, in mind. You will need a bound reference book for your SACs and exams, and it would be a waste of time for you to write your own notes. I want you to use this book as your reference book, but understand that you may also want to add your own notes. This space will allow you to do so.

For now, begin reading, writing and working through this student guide. If you can absorb all of the concepts, tips and suggestions, I am sure that you will soon be on a path towards mathematical success.

I wish you all the best for your studies.

Derrick Ha
September 2016

About the Author



Over the past nine years, Derrick has established himself as an author, tutor and lecturer in senior VCE mathematics. Since founding Derrick Ha Student Coaching Services in 2008, Derrick and his team have assisted thousands of students with their VCE and helped them to achieve their goals.

Derrick's unique teaching style is drawn from his personal experiences of the VCE and his active extracurricular involvement. In 2007, Derrick attained the top ENTER (ATAR) score of 99.95, with the following results:

	Subject	**Raw Study Score**	**Year**
Top 4 Subjects	English Language	50	2007 (Year 12)
	Specialist Mathematics	50 (scaled to 54.2)	2007
	Accounting	50	2006 (Year 11)
	Mathematical Methods	50	2005 (Year 10)
5th Subject	Chemistry	50 (5.0 increment)	2006
6th Subject	University Mathematics	High Distinction (5.5 increment)	2007
		Aggregate = 214.7	
Extra 7th Subject	English	48	2007

Derrick’s achievements also extend outside of the VCE. He had many successes in mathematics competitions and, in 2007, achieved a perfect score in the Australian Mathematics Competition. He was awarded a gold medal and the B H Neumann Certificate for being the only Senior student in Australasia to achieve this perfect score. He was also an invited attendee of four training selection schools for the Australian Mathematical Olympiad Team. In both 2004 and 2005, Derrick was awarded a Diploma by the Russian Academy of Sciences for his accomplishments in the International Tournament of Towns Mathematics Competition.

Derrick also has extensive experience in public speaking and mathematics coaching. He is a motivational speaker for high schools and community organisations, and was a lecturer for state-wide end-of-year revision lectures in both Mathematical Methods CAS and Specialist Mathematics. Derrick also works as a mathematics tutor, and has previously volunteered to teach English and mathematics to Sudanese immigrants.

His accomplishments as an orator include being a speaker in the team that reached the DAV Debating State Finals in four separate years. He also experienced success in mock-law courts, as a speaker in the legal team that won the State Mooting Titles in the 2007 Bond University Mooting Competition.

In his VCE year, Derrick was the School Vice-Captain of Haileybury College and the Firsts Team Badminton Captain. He was a recipient of the VCE Premier’s Award Top All-Round Achiever, the Australian Student Prize and the Monash University Prize for Academic Excellence in Year 11.

In 2013, Derrick completed his medical training at the University of Melbourne. He was the recipient of the National Medicine Full Scholarship and won multiple Dean’s Honours Awards. Derrick currently works as a full-time medical doctor.

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Sample – Theory

This is an example of the notes that are provided in the book. These are designed to explain each topic in a practical and easy-to-understand manner. Notes are provided for every topic on the study design.

Population Proportion vs Sample Proportion

The concept of ‘population proportion’ and ‘sample proportion’ is best explained with an example:

> Consider a very large group of 3000 high school students. If 1000 of these people have black hair, then this would mean that the proportion of the population with black hair would be one-third (i.e. 1000 divided by 3000). However, if you were counting this group of high school students in real-life, then you would probably struggle to count every single person (since there are so many students to count).
>
> As a compromise, we might instead choose to count a random sample of 60 students from the 3000 total students. If we took this sample of 60 students, then we would expect to have 20 black hair students. However, as you can probably imagine, we might find that we have 19 or 21 black hair students or some other number entirely.
>
> If we had counted 19 students with black hair out of 60, then we would think that the proportion of students with black hair is $\frac{19}{60}$. This would be called the sample proportion.
>
> In this above example, we have found that population proportion is $\frac{1}{3}$. We can use the symbol p to represent this population proportion. We also found that the sample proportion is $\frac{19}{60}$. We use the symbol $\hat{p}$ to represent the sample proportion.
>
> You should appreciate that the sample proportion is dependent on how many students are in your sample had black hair. In other words, if we repeated the task the next day and took another random sample of 60 students, then we may end up with a different sample proportion. In comparison, the population proportion will never change because we always have the same population of 3000 students and 1000 of them will always have black hair.

Summary:

- Population proportion is constant
- Sample proportion will vary from sample to sample.

Sample – Hand-Written Solution

This is an example of the hand-written solutions provided in the book. These are designed to illustrate Derrick's thought process, as well as showing how you can set out your own working in the VCAA exam.

Question:

Solve for x, where $\log_2(2x+1)+1 \leq 5$.

Hand-Written Solution:

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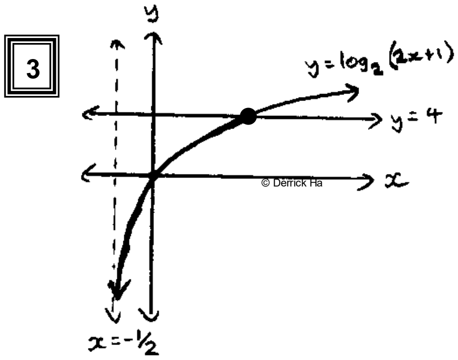

1 $\log_2(2x+1) + 1 \leq 5$

2 $\log_2(2x+1) \leq 4$

Consider the graphs of $y = 4$ and $y = \log_2(2x+1)$:

4 Find intersection:

$$4 = \log_2(2x+1)$$

$$2^4 = 2x+1$$

$$2x = 15$$

5 $x = 15/2$

6 $\therefore$ From graph: $-\frac{1}{2} < x \leq \frac{15}{2}$

(ie. due to asymptote)

1 – Firstly, try to rearrange the inequality as if you were solving for x. Here, we do this by subtracting "1" from both sides.

Stop when you reach the stage where you need to convert from logarithms to exponentials (or vice versa). We reach this stage at (2)

2 – Here, in order to solve for x, we need to convert from logarithms to exponentials. This is problematic because after we do this, it is difficult to determine which way the inequality sign should point. Furthermore, if we simply converted to exponentials, we would be disregarding the effects of any asymptotes (we will see later that asymptotes change the answer).

To avoid these problems and to simplify the question, we draw a graph.

3 – Here, we sketch the graphs of y = LHS and y = RHS. These graphs are useful because now the inequality can be seen visually. That is, from (2), we know that the logarithm needs to be less than or equal to 4. On the graph, this is indicated by the thickened region of the log graph. Note the closed circle at the intersection of the two graphs – it is closed because we have a less than **<u>or equals to</u>** sign in the inequality.

Look at the thickened part of the log graph. We can see that it corresponds to a set of x-values that are to the 'left' of the intersection point. In other words, our final answer for the entire question is going to be $x < something$.

We need to find the value of this 'something' – i.e. we need to find the x-value at the intersection of the two graphs.

4 – Find the intersection point – note how we do not need to worry about any inequalities when finding this point (this is why the entire method is simpler).

5 – This is the x-value at the intersection point. From the graph, we see that the thickened part is to the 'left' of this point. Therefore $x < 15/2$.

6 – One significant benefit of drawing the graph is that we can see the asymptote at x = -½. Therefore, we also know that $x >$ -½. That is, we cannot have x less than -½ because this would involve taking the logarithm of a negative number, which would be mathematically incorrect.

Sample – Exam Question

This is an example of a question from one of the four trial exams in the book. Although not shown in this sample, the book also contains hand-written solutions for all of the exams in order to demonstrate how to set out working in a VCE exam.

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Question 2

Frank is a dentist. He uses an ultrasonic, vibrating drill to clean and polish teeth. The drill operates as follows:

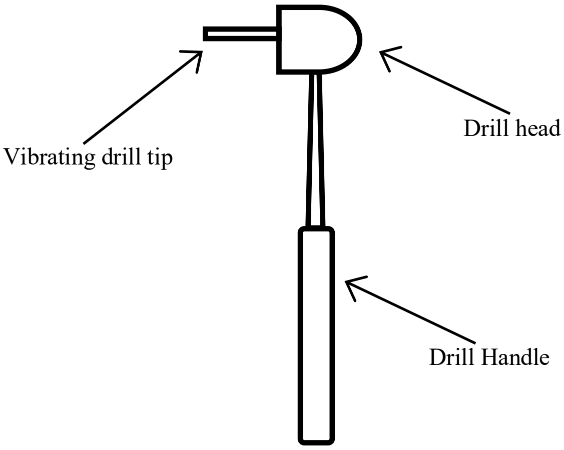

Figure 1 – The Dentist Drill

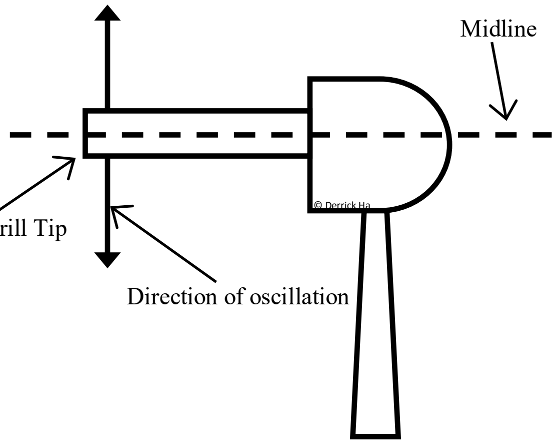

Figure 2 – Close-Up of Drill Head Showing Vibrating Drill Tip

The drill is used to remove debris from the surface of a tooth. It achieves this by using a 'vibrating drill tip', which rapidly oscillates 'back and forth' to scratch the debris away from the tooth. The 'vibrating drill tip' is attached to a 'drill head', which is in turn attached to a 'drill handle' for Frank to hold.

The vibration of the drill tip can be modelled by a mathematical formula. It is known that the very end of the drill tip moves straight up and down in a repetitive motion, as shown in Figure 2. The location of this tip relative to the midline is represented by the following equation:

$H(t) = k\sin(at), \quad k > 0, a > 0$ Where t is in microseconds and H represents position in micrometres relative to the midline.

a. Given that $k = 70$ and $a = \frac{\pi}{60}$, calculate the initial position of the end of the drill tip, as well as its position after 1 millisecond.

2 marks

In order to effectively remove debris, the end of the drill tip must:

- Oscillate at least 10000 times per second (one period of the graph equals one oscillation)
- Be displaced either side of the midline by at least 45 micrometres during each oscillation.

b. Given that a particular dentist drill is able to effectively remove debris, find the range of possible values for k and a.

2 marks

Frank's drill tip is able to oscillate every 60 microseconds and deviates from the midline by a maximum of 50 micrometres.

c. Show that the equation for Frank's drill tip is $H(t) = 50\sin\left(\frac{\pi}{30}t\right)$.

1 mark

A particular type of tooth stain, known as the 'Dubi Stain' is only able to be removed if the drill tip is travelling at a speed of at least 3.5 micrometres per microsecond and if this speed is maintained for a continuous period of 14 microseconds. It is known that the velocity is equal to the derivative of position with respect to time and that speed is the magnitude of velocity.

d.

i. Determine whether Frank's drill is **fast enough** to remove a 'Dubi Stain'.

2 marks

ii. Hence, determine whether Frank's drill is able to remove a 'Dubi Stain'.

3 marks

Frank's trainee, Melissa, is practising on a dummy with one of Frank's old dentist drills. This second drill is plugged into the mains power supply on the wall. Unfortunately, as a result of its old age, this second drill malfunctions, with erratic oscillation of the end of its drill tip.

As a keen mathematician, Melissa investigates this erratic oscillation and determines that the drill is behaving according to the equation:

$$D(t) = 50\sin\left(\frac{\pi}{30}t\right) + 30\sin\left(\frac{\pi}{150}t\right) + 80$$

where t is in microseconds and D represents position in micrometres relative to the midline.

e. For Melissa's drill, state the period of $D(t)$ and write down the maximum distance of the end of the drill tip from the midline.

1 mark

When the graph of D is plotted against t, the area under the graph represents the power required by the drill. Each square unit under the graph represents one unit of power.

If the drill is to operate on battery power, it must not draw more than 10000 units of power over any 0.3 millisecond period.

f. Determine the exact amount of power that Melissa's Drill requires over a 0.3 millisecond period, and hence state whether Melissa's Drill is able to operate using battery power.

3 marks

Total 14 marks